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Coherence Measurements in an Axisymmetric Wake

J. B. ROBERTS*

University of Sussex, Falmer, Brighton, Sussex, England

1. Introduction

STUDIES of the structure of the wake behind axisymmetric blunt-based bodies extend over a period of many years. Much of the early work concentrated on low Reynolds number flow (e.g., see Ref. 1) and was mainly confined to flow visualization studies. A notable exception is the work of Winny[2] who examined concurrent signals from two hot wires in an attempt to detect the wake structure behind a sphere, at high Reynolds numbers.

More recently a number of experimental studies of axisymmetric wake flows at high Reynolds numbers have been published[3–9] and various aerospace problems which have arisen in the last few years have stimulated renewed interest in this aspect of bluff-body aerodynamics.[9]

Previous studies have established, by spectral analysis of single hot wire signals, that a definite periodicity exists in axisymmetric wake flows behind blunt-based bodies but the actual wake structure, for Reynolds numbers above a few hundred, is still unknown. It has been suggested by Calvert[7] and others[5] that the analysis of concurrent signals from two hot wires, and in particular the cross-spectral analysis of such signals, is likely to yield considerable information on the wake structure. The purpose of this Note is to report the results of a series of such measurements. This approach is, of course, in the spirit of Winny's early work but advantage can now be taken of developments which have occurred in hot wire instrumentation and in the statistical analysis of random signals. Winny's work was restricted to a visual examination of concurrent hot wire signals and this enabled only very rough estimates of phase relationships to be made. In contrast, in the present work, the very powerful techniques of digital spectral and cross-spectral analysis are employed to yield precise quantitative information on the frequency dependence of the correlation and phase relationships between hot wire signals.

2. Experimental Equipment

The experimental measurements were performed in a closed circuit wind tunnel having a test section measuring 3 ft square by 14.5 ft long. The background turbulence intensity was less than 0.5% throughout the test section.

A 3-in.-diam stainless steel disk, with a sharp edged upstream face, was mounted on wires, normal to the flow, at a position 6 ft downstream of the inlet to the test section, and in the center of the tunnel. By observing the reflection of a pair of crossed wires in the highly polished upstream face of the disk, with a telescope, this face was accurately aligned to be normal to the centerline of the tunnel test section.

Near the station where measurements were to be taken a $\frac{1}{2}$-in. diam rod was fixed across the tunnel to support two hot wire probes. The hot wire systems used were commercial Disa constant temperature anemometer 55AO1 sets and the probes used were standard Disa miniature hot wires with active elements of 0.5-μm-diam platinum plated tungsten wire, 1.25 mm long.

The wind-tunnel speed was monitored by a Betz manometer, which indicated the difference between static pressures in the test section and the settling chamber. This had previously been calibrated against a pitot-static probe in the test section.

3. Data Analysis

Let $u(t)$ denote the variation of the longitudinal velocity component with time recorded by one hot wire probe. Assuming $u(t)$ is a realization of a stationary random process, with mean m, the power spectrum of $u(t)$ is defined as

$$S(f) = 2\int_{-\infty}^{\infty} w(\tau)\cos(2\pi f\tau)\,d\tau$$

where $w(\tau)$, the covariance function of $u(t)$ is given by

$$w(\tau) = E\{[u(t)-m][u(t+\tau)-m]\}^{\dagger}$$

For two hot wire signals, $u_1(t)$ and $u_2(t)$, with means m_1 and m_2, and power spectra $S_1(f)$ and $S_2(f)$, respectively, the cross-spectrum is defined as

$$S_{12}(f) = 2\int_{-\infty}^{\infty} w_{12}(\tau)\exp(-i2\pi f\tau)\,d\tau$$

where

$$w_{12}(\tau) = E\{[u_1(t)-m_1][u_2(t)-m_2]\}$$

The nondimensional coherence function, C, is defined as

$$C = S_{12}(f)/[S_1(f)S_2(f)]^{1/2} = C_r + iC_i$$

Power spectra and cross-spectra of the hot wire signals, together with the related coherence function, were obtained from the spectral analysis system described by Roberts and Surry.[10] The data was analyzed in real time, using analog lines to a remote analog-to-digital converter and computer. Tests proved that the analog lines introduced negligible degradation of data, over the frequency range considered here.[10]

The conventional Blackman and Tukey correlation technique was used to generate the power and cross-spectral estimates, correlation computations being performed between successive samples.

4. Discussion of Results

Throughout the experiments the tunnel mean wind speed U was maintained at 50 fps, giving a Reynolds number based on disk diameter of 7.8×10^4. A variety of mean flow and intensity traverses were made downstream of the disk to check the symmetry of the wake.

a. Single probe results

Initially, the periodicity in the wake was investigated by performing a power spectrum analysis on the signal from a single hot wire probe, at various positions in the wake. In view of the uncertainties in hot wire measurements in high intensity turbulence, measurements were confined to probe positions at least nine diameters downstream of the disk ($X/D = 9$); here the turbulence intensity was everywhere less than 10%.

A definite peak was observed in the power spectrum of the probe signal, at all probe positions in the wake except those directly on the disk axis of symmetry. The frequency f of this peak was independent of probe position, with a Strouhal number $S = fD/U$ of 0.135, in agreement with the observations of Calvert.[7] On traversing the probe radially outwards, at a fixed X/D, the height of the spectral peak was found to increase

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* Lecturer in Mechanical Engineering, School of Applied Sciences.

† $E\{\ \}$ denotes the expectation, or ensemble averaging, operator.

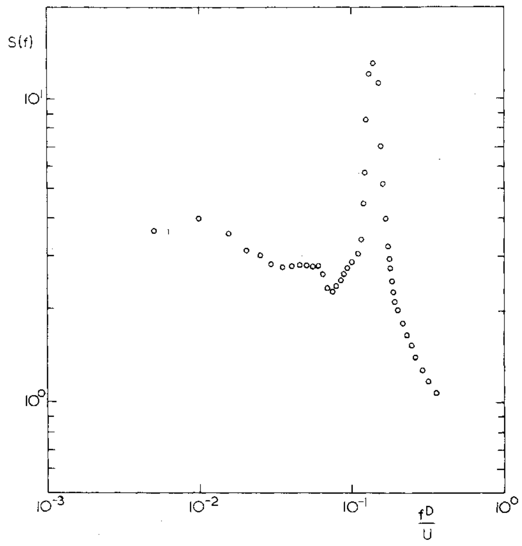

Fig. 1 Power spectra from a single hot wire probe at $X/D = 9$ and $r/D = 0.83$.

from zero to a maximum value, and then to decrease gradually to zero again. The peak height was also found to decrease with increasing axial probe distance from the disk. The largest spectral peak was observed at the minimum distance $X/D = 9$, and a radial distance of 0.83 disk diameters ($r/D = .83$); the appropriate power spectrum for this case, shown in Fig. 1, was computed from 234,300 samples, taken at the rate of 140/sec.

b. Coherence measurements

For the purpose of coherence measurements two hot wire probes were used, these being confined to lie in a plane normal to the axis of symmetry, at an axial distance $X/D = 9$, and were further confined to have the same radial displacement of $r/D = 0.83$. Coherence measurements were taken at angular separations, θ, of the probes of 20°, 45°, 65°, 90°, 110°, 135°, 155°, and 180°. In each case coherence estimates were computed from 234,300 samples, on each signal, taken at a rate of 140/sec. For each angular separation the experiment was repeated for various positions on the chosen circle, to check the symmetry of the results.

For all the θ's considered, the imaginary part of the coherence was found to be effectively zero over the whole frequency range considered. This of course implies that at frequencies where the real part of the coherence is significantly nonzero the signals are either in phase or 180° out of phase. Typical results for the variation of the real part of the coherence, C_r, with frequency are shown in Fig. 2. It is observed that at low frequencies the coherence has a high positive value for small θ, falls progressively to about -0.6 as θ increases to 90°, increases again when θ exceeds 90°, and finally attains a value of about 0.6 when $\theta = 180°$. At $S = 0.135$, corresponding to the peak in the power spectra, the coherence falls monotonically with increasing θ, being approximately zero at $\theta = 90°$ and about -0.8 at $\theta = 180°$. At higher frequencies the coherence decreases fairly steadily to zero with increasing θ, with a shallow negative lobe. These trends are clearly shown in Fig. 3, which shows the variation of C_r with θ for three frequencies; this graph was obtained by averaging a number of results of the type shown in Fig. 2.

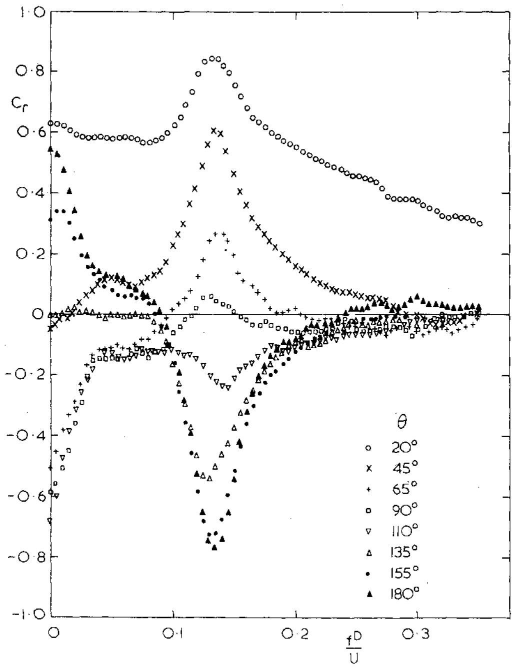

Fig. 2 Variation of C_r with frequency, for various angular separations. $X/D = 9$ and $r/D = 0.83$, for both probes.

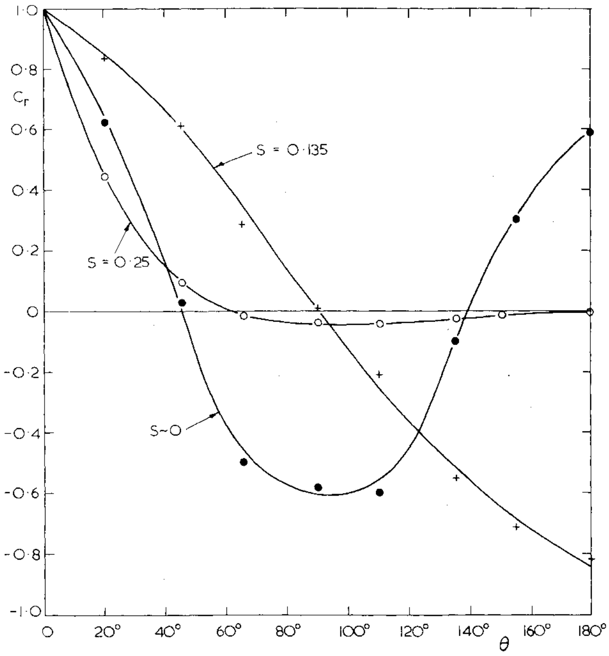

Fig. 3 Variation of C_r with θ, for three frequencies. $X/D = 9$ and $r/D = 0.83$, for both probes.

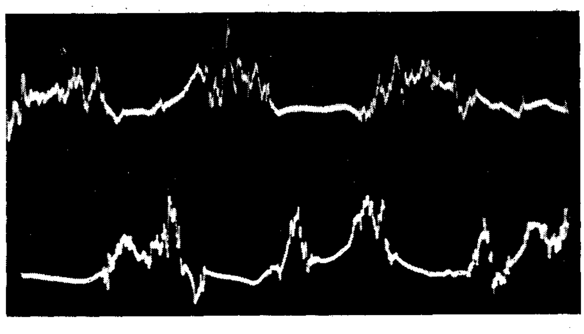

Fig. 4 Typical concurrent hot wire signals from two hot wires, at an angular separation of 180°. $X/D = 9$ and $r/D = 0.83$.

A visual examination of the hot wire signals clearly revealed the highly intermittent character of the wake turbulence. At the probe locations chosen, the intermittency factor was about 0.5 and the average frequency of occurrence of the turbulent bursts corresponded to the frequency of the peak in the power spectrum. The photograph of concurrent hot wire signals from two probes at $\theta = 180°$, shown in Fig. 4, illustrates the tendency of the intermittent bursts to be antiphase, at this value of θ. This behavior is reflected by the high negative coherence found at $\theta = 180°$ and $S = 0.135$, as shown in Fig. 3, and is consistent with the notion of a simple "flapping" motion of the turbulent core, the core tending to engulf the hot wire probes alternately. The coherence results for low frequencies, shown in Fig.3, are consistent wih a slow, random variation in the orientation of such a flapping motion.

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Elastic Wave Surfaces in Heterogeneous Anisotropic Plates

A. S. D. WANG* AND D. L. TUCKMANTEL†

Drexel University, Philadelphia, Pa.

Introduction

ELASTIC wave surfaces due to impact on anisotropic plates have recently been investigated by Moon.[1] In the analysis, Moon employed an effective modulus plate theory and considered a class of specially laminated fiber-reinforced composite plates, which uncouples the transverse, bending and extensional displacements; a wave surface approach was used to describe the propagation of plane acceleration waves. The present Note is concerned with elastic wave surfaces in plates that are composed of layers possessing arbitrary anisotropy. In this case a severe coupling exists among the shear bending, twisting and extensional effects, resulting in simultaneously coupled wave surfaces in the plane of the plate. We follow a general laminated plate theory[2] and apply a control volume approach for the analysis.[3]

Explicit solutions for the coupled wave surfaces and their velocities are obtained. Several numerical problems involving laminated fiber-reinforced composite plates are presented and their unique features discussed.

Analysis

Let us consider a thin laminated plate of thickness h (Fig. 1). The laminae comprising the plate are assumed to be individually homogeneous and anisotropic. Thus the inhomogeneity of the plate occurs only in the thickness direction. We shall consider a plane wave front which originates at an arbitrary point in the plate, for convenience let us say at the origin of the (x, y, z) system, and propagates in the x, y plane. At any given instant, the wave surface is denoted by S. Let $\bar{n}$ be the normal of S at a point A on S, and let $\bar{s}$ be the tangent of S at the same point. The wave surface S is assumed to propagate in the direction $\bar{n}$ with a constant speed $c(\bar{n})$.

Let u, v, w, ψ_n and ψ_s be the displacements, referred to the local coordinates (n, s, z). Following the laminated plate theory,[2] their forms are assumed as

$$u = u^o(n,s,t)+z\psi_n(n,s,t), \quad v = v^o(n,s,t)+z\psi_s(n,s,t), \quad w = w^o(n,s,t) \tag{1}$$

The kinematic (continuity) conditions across S at any given point on S require (see Ref. 3)

$$[u^o] = [v^o] = [w^o] = [\psi_n] = [\psi_s] = 0$$

$$[u_{,n}{}^o, v_{,n}{}^o, w_{,n}{}^o, \psi_{n,n}, \psi_{s,n}] = (1/c)[u_{,t}{}^o, v_{,t}{}^o, w_{,t}{}^o, \psi_{n,t}, \psi_{s,t}]$$

$$(u, v, w, \psi_n, \psi_s)_{,s} = 0$$

where [ ] represent a discontinuity of the enclosed quantity across S.

With these conditions, the plate constitutive relations, when referred to local coordinates (n, s, z) yield

$$\begin{bmatrix} N_n \\ N_{ns} \\ M_n \\ M_{ns} \\ Q_n \end{bmatrix} = \begin{bmatrix} A_{11} & A_{16} & B_{11} & B_{16} & A_{15} \\ A_{16} & A_{66} & B_{16} & B_{66} & A_{56} \\ B_{11} & B_{16} & D_{11} & D_{16} & B_{15} \\ B_{16} & B_{66} & D_{16} & D_{66} & B_{56} \\ A_{15} & A_{56} & B_{15} & B_{56} & A_{55} \end{bmatrix} \begin{bmatrix} u_{,n}{}^o \\ v_{,n}{}^o \\ \psi_{n,n} \\ \psi_{s,n} \\ w_{,n}{}^o \end{bmatrix} \tag{2}$$

where N, Q, M and the constants A, B and D.s take the same meaning as those defined in Ref. 2 only in this case all quantities are referred to the local coordinates (n, s, z).

The dynamic relations across the wave surface are established by defining a control volume which is located on S at point A, as shown in Fig. 1. Since the control volume moves with the wave front, an observer fixed with it, sees a normal influx of mass entering with a speed $U_1 = c - u_{1,t}$, and a normal efflux of mass leaving with a speed $U_2 = c - u_{2,t}$. Thus the steady-state conservation of mass for the control volume yields

$$\int (\rho_2 U_2 - \rho_1 U_1)\, dz = 0 \tag{3}$$

where ρ is the mass density of the material and the subscripts 1 and 2 refer to the properties ahead of and behind the wave front S, respectively. The integral is carried over the thickness of the plate $[(-h/2), (h/2)]$.

It is noted that condition (3) is satisfied by a more restrictive condition, resulting from the classical thin plate assumptions, namely $\rho_2 U_2 = \rho_1 U_1$. The force and moment resultants acting upon the control volume must satisfy the equations of balanced momenta. These are

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* Associate Professor of Applied Mechanics. Member AIAA.

† Presently at Dyna East Corporation, Philadelphia, Pa.